Lecture 9
Part 1: Effect of image motion on image quality
Part 2: Detectors and signal to noise ratio
Part 3: Class projects

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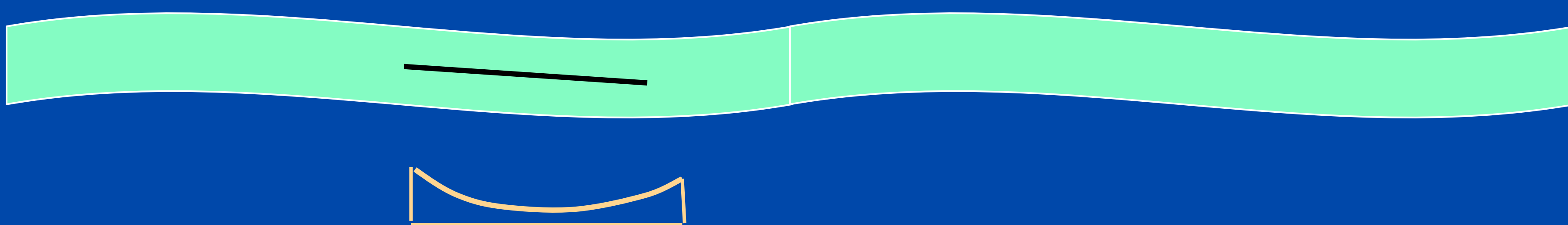
February 9, 2016

Part One: Image motion and its effects on Strehl ratio

- Sources of image motion:
  - Telescope shake due to wind buffeting (hard to model *a priori* – depends on telescope, dome, …)
  - Atmospheric turbulence
- Image motion due to turbulence:
  - Sensitive to atm. inhomogenities > telescope diam. D
  - Hence reduced if "outer scale" of turbulence is ≤ D



Long exposures, no AO correction

$$FWHM(\lambda) = 0.98 \frac{\lambda}{r_0}$$

- "Seeing limited": Units are radians
- Seeing disk gets slightly smaller at longer wavelengths:

  $FWHM \sim \lambda / \lambda^{-6/5} \sim \lambda^{-1/5}$
- For completely uncompensated images, wavefront error

  $\sigma^2_{uncomp} = 1.02 \, (D / r_0)^{5/3}$

Correcting tip-tilt has relatively large effect, for seeing-limited images

- For completely uncompensated images

$$\sigma^2_{uncomp} = 1.02\ (D / r_0)^{5/3}$$

- If image motion (tip-tilt) has been completely removed

$$\sigma^2_{tiltcomp} = 0.134\ (D / r_0)^{5/3}$$

(Tyson, Principles of AO, eqns 2.61 and 2.62)

- Removing image motion can (in principle) improve the wavefront variance of an uncompensated image by a factor of 10

- Origin of statement that "Tip-tilt is the single largest contributor to wavefront error"

But you have to be careful if you want to apply this statement to AO correction

- If tip-tilt has been <u>completely</u> removed

$$\sigma^2_{tiltcomp} = 0.134 \; (D \; / \; r_0 \;)^{5/3}$$

- But typical values of $(D \; / \; r_0 \;)$ are 10 - 50 in the near-IR
  - Keck, D=10 m, r_0 = 60 cm at λ=2 μm, (D/r_0) = 17

$$\sigma^2_{tiltcomp} = 0.134 \; (\; 17 \;)^{5/3} \; \sim \; 15$$

so wavefront phase variance is >> 1

- Conclusion: if $(D/r_0) \gg 1$, removing tilt alone won't give you anywhere near a diffraction limited image

Scaling of image motion due to turbulence (review)

- **Mean squared deflection angle due to image motion: independent of λ and ~ $D^{-1/3}$**

$$\sigma_\alpha^2 = 0.182 \left(\frac{D}{r_0} \right)^{5/3} \left(\frac{\lambda}{D} \right)^2 \quad radians^2$$

- **But relative to Airy disk (diffraction limit), image motion gets <u>worse</u> for larger D and smaller wavelengths:**

$$\frac{\sigma_\alpha}{(\lambda / D)} = 0.43 \left(\frac{D}{r_0} \right)^{5/6} \propto \frac{D^{5/6}}{\lambda}$$

Typical values of image motion

- **Keck Telescope: D = 10 m, r_0 = 0.2 m, λ = 2 microns**

$$\sigma_\alpha = 0.43 \left(\frac{D}{r_0}\right)^{5/6} \left(\frac{\lambda}{D}\right) = 0.43 \left(\frac{10m}{0.2m}\right)^{5/6} \left(\frac{2\times 10^{-6} m}{10m}\right) = 0.45 \text{ arc sec}$$

$$\frac{\lambda}{D} = 0.04 \text{ arc sec} \qquad \text{(Recall that 1 arcsec} = 5\ \mu rad)$$

- **So in theory at least, rms image motion is > 10 times larger than diffraction limit, for these numbers.**

What maximum tilt must the tip-tilt mirror be able to correct?

- For a Gaussian distribution, probability is 99.4% that the value will be within ± 2.5 standard deviations of the mean.

- For this condition, the peak excursion of the angle of arrival is

$$\alpha_{peak} = \pm \underset{\substack{\uparrow \\ 2.5\,\sigma}}{1.07} \left(\frac{D}{r_0} \right)^{5/6} \left(\frac{\lambda}{D} \right) \text{ radians } \approx 2 \text{ arc sec}$$

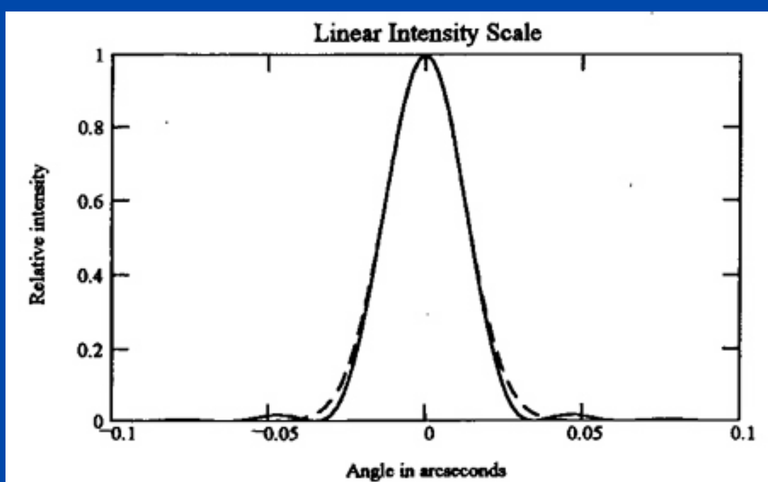
- Note that peak angle is independent of wavelength

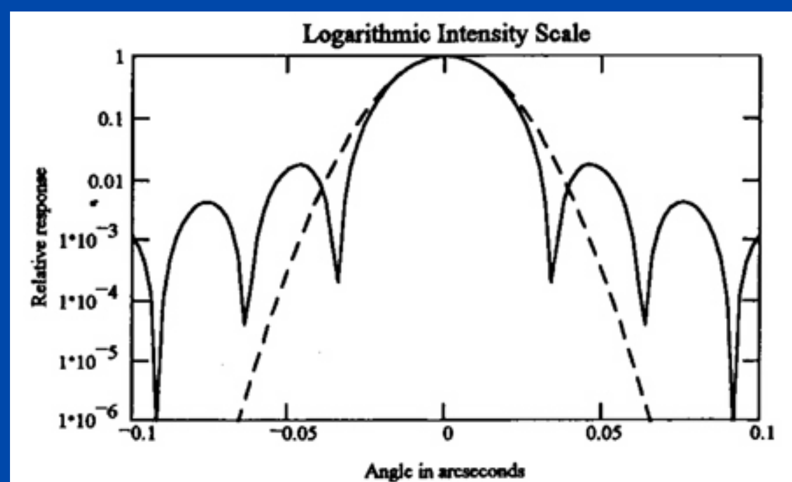
Use Gaussians to model the effects of image motion on image quality

- Model the diffraction limited core as a Gaussian:

$$G(x) = \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{x^2}{2\sigma^2} \right)$$

- A Gaussian profile with standard deviation $\sigma_A = 0.44\left(\frac{\lambda}{D}\right)$ has same width as an Airy function





Tilt errors spread out the core

- Effect of random tilt error σ_α is to spread each point of image into a Gaussian profile with standard deviation σ_α
- If initial profile has width σ_A then the profile with tilt has width $\sigma_T = (\sigma_\alpha^2 + \sigma_A^2)^{1/2}$ (see next slide)

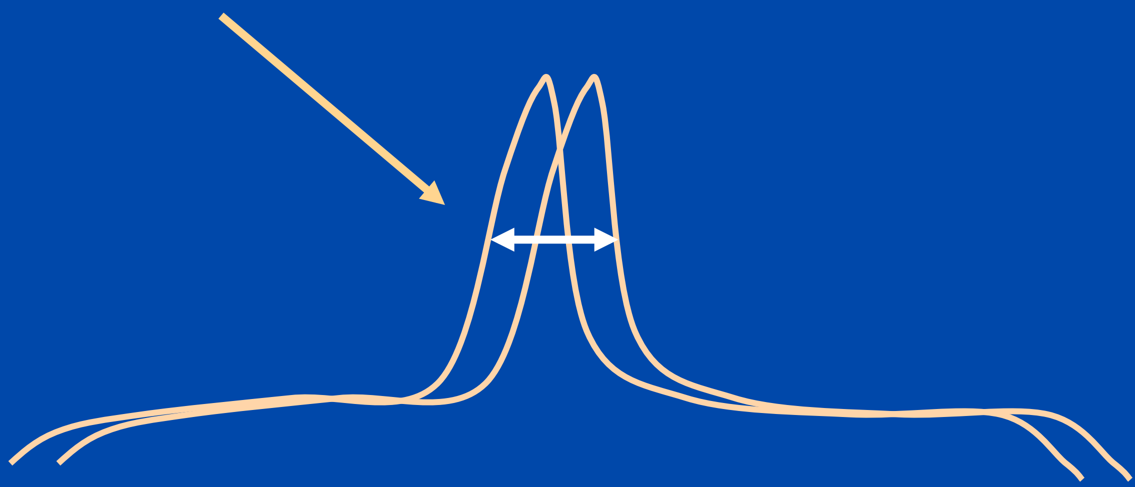


Image motion reduces peak intensity

- Conserve flux:
  - Integral under a circular Gaussian profile with peak amplitude A_0 is equal to $2\pi A_0 \sigma_A^2$
  - Image motion keeps total energy the same, but puts it in a new Gaussian with variance $\sigma_T^2 = \sigma_A^2 + \sigma_\alpha^2$
  - Peak intensity is reduced by the ratio

$$F_T = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_\alpha^2} = \frac{1}{1 + (\sigma_\alpha / \sigma_A)^2}$$

Tilt effects on point spread function, continued

- Since $\sigma_A = 0.44\ \lambda\ /\ D$, peak intensity of the previously diffraction-limited core is reduced by

$$F_T = \frac{1}{1 + \left(D / 0.44\lambda\right)^2 \sigma_\alpha^2}$$

- Diameter of core is increased by $F_T^{-1/2}$
- Similar calculations for the halo: replace *D* by r_0
- Since $D >> r_0$ for cases of interest, effect on halo is modest but effect on core can be large

Typical values for Keck Telescope, if tip-tilt is not corrected

- Core is strongly affected at a wavelength of 1 micron:

$$\sigma_\alpha \cong 0.5 \text{ arcsec}, \lambda / D = 0.02 \text{ arcsec}, F_T = \frac{1}{1 + (\sigma_\alpha / \sigma_A)^2} \approx 0.002$$

  - Core diameter is increased by factor of $F_T^{-1/2}$ ~ 23

- Halo is much less affected than core:
  - Halo peak intensity is only reduced by factor of 0.93
  - Halo diameter is only increased by factor of 1.04

Effect of tip-tilt on Strehl ratio

- Define S_c as the peak intensity ratio of the core alone:

$$S_c = \frac{\exp(-\sigma_\phi^2)}{1 + (D/0.44\lambda)^2 \sigma_\alpha^2}$$

- Image motion relative to Airy disk size *1.22 λ / D* :

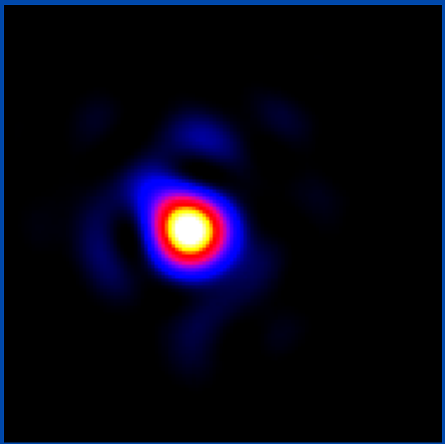
$$\frac{\sigma_\alpha}{(1.22\lambda / D)} = 0.36 \left[\frac{\exp(-\sigma_\phi^2)}{S_c} - 1 \right]^{-1/2}$$

- Example: To obtain Strehl of 0.8 from tip-tilt only (no phase error at all, so σ_φ = 0), σ_α = *0.18 (1.22 λ / D)*
  - Residual tilt has to be w/in 18% of Airy disk diameter

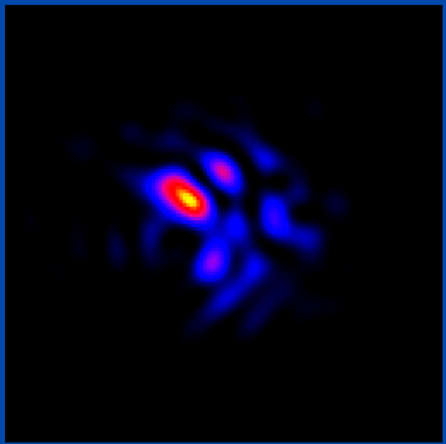
Effects of turbulence depend on size of telescope

- Coherence length of turbulence: r_0 (Fried's parameter)
- For telescope diameter D < (2 - 3) x r_0 :
  Dominant effect is "image wander"
- As D becomes >> r_0 :
  Many small "speckles" develop

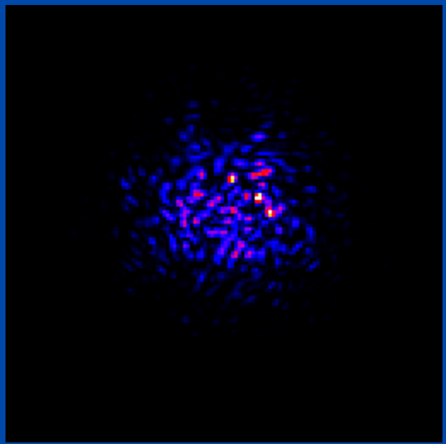
- Computer simulations by Nick Kaiser: image of a star, r_0 = 40 cm



D = 1 m



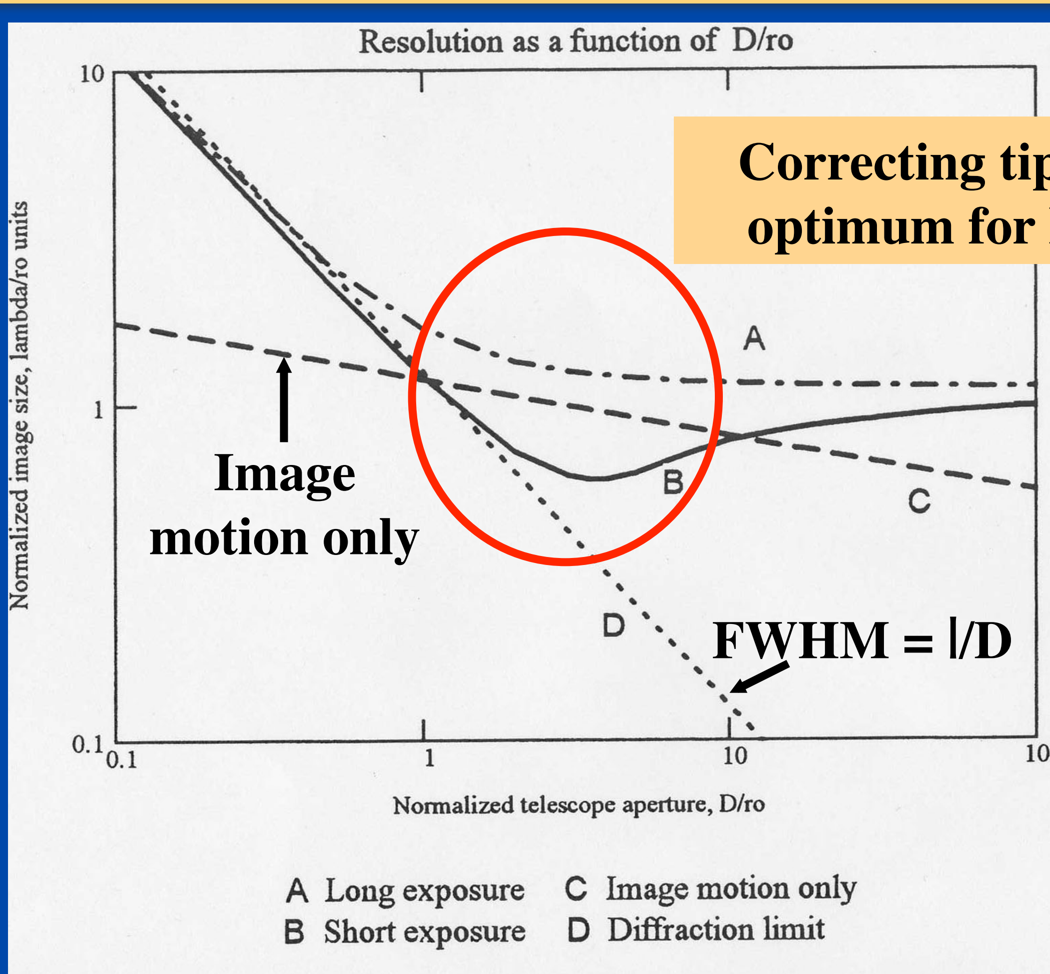
D = 2 m



D = 8 m

Effect of atmosphere on long and short exposure images of a star

Hardy p. 94



Correcting tip-tilt <u>only</u> is optimum for $D/r_0 \sim 1$ - 3

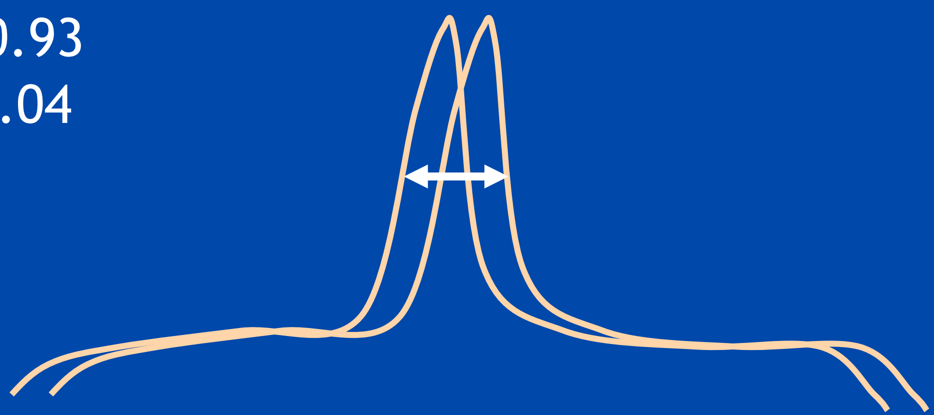
Vertical axis is image size in units of λ / r_0

Summary, Image Motion Effects (1)

- Image motion
  - Broadens core of AO PSF
  - Contributes to Strehl degradation differently than high-order aberrations
  - Effect on Strehl ratio can be quite large: crucial to correct tip-tilt

Summary, Image Motion Effects (2)

- Image motion can be large, <u>if</u> not compensated
  - Keck, λ = 1 micron, σ_α = *0.5 arc sec*
- Enters computation of overall Strehl ratio differently than higher order wavefront errors
- Lowers peak intensity of core by F_c^{-1} ~ 1 / 0.002 = 500 x
- Halo is much less affected:
  - Peak intensity decreased by 0.93
  - Halo diameter increased by 1.04



How to correct for image motion

- Natural guide star AO:
  - From wavefront sensor information, filter out all higher order modes, left with overall tip-tilt
  - Correct this tip-tilt with a “tip-tilt mirror”
- Laser guide star AO:
  - Can use laser to correct for high-order aberrations but not for image motion (laser goes both up and down thru atmosphere, hence moves relative to stars)
  - So LGS AO needs to have a so-called “tip-tilt star” within roughly an arc min of target.
  - Can be faint: down to 18-19th magnitude will work

Implications of image motion for AO system design

- Impact of image motion will be different, depending on the science you want to do
- Example 1: Search for planets around nearby stars
  - You can use the star itself for tip-tilt info
  - Little negative impact of image motion smearing
- Example 2: Studies of high-redshift galaxies
  - Sufficiently bright tip-tilt stars will be rare
  - Trade-off between fraction of sky where you can get adequate tip-tilt correction, and the amount of tolerable image-motion blurring
    - High sky coverage → fainter tip-tilt stars farther away

Part 2: Detectors and signal to noise ratio

- Detector technology
  - Basic detector concepts
  - Modern detectors: CCDs and IR arrays

- Signal-to-Noise Ratio (SNR)
  - Introduction to noise sources
  - Expressions for signal-to-noise
    - Terminology is not standardized
    - Two Keys: 1) Write out what you're measuring.
2) Be careful about units!
    - Work directly in photo-electrons where possible

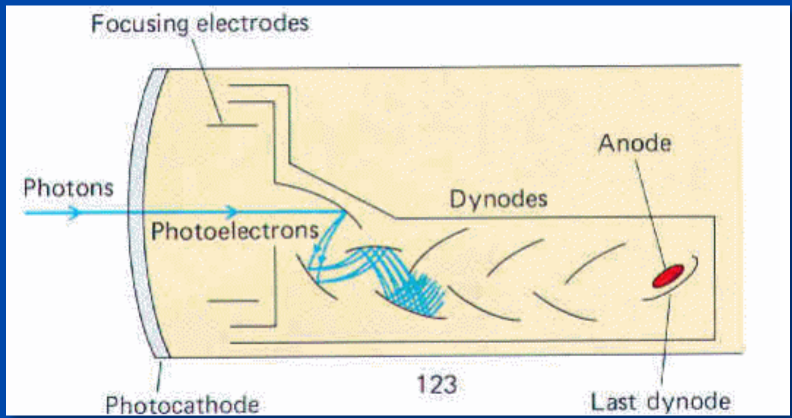
References for detectors and signal to noise ratio

- Excerpt from "Electronic imaging in astronomy", Ian. S. McLean (1997 Wiley/Praxis)
- Excerpt from "Astronomy Methods", Hale Bradt (Cambridge University Press)
- Both are in Reader

Early detectors: photographic plates, photomultipliers

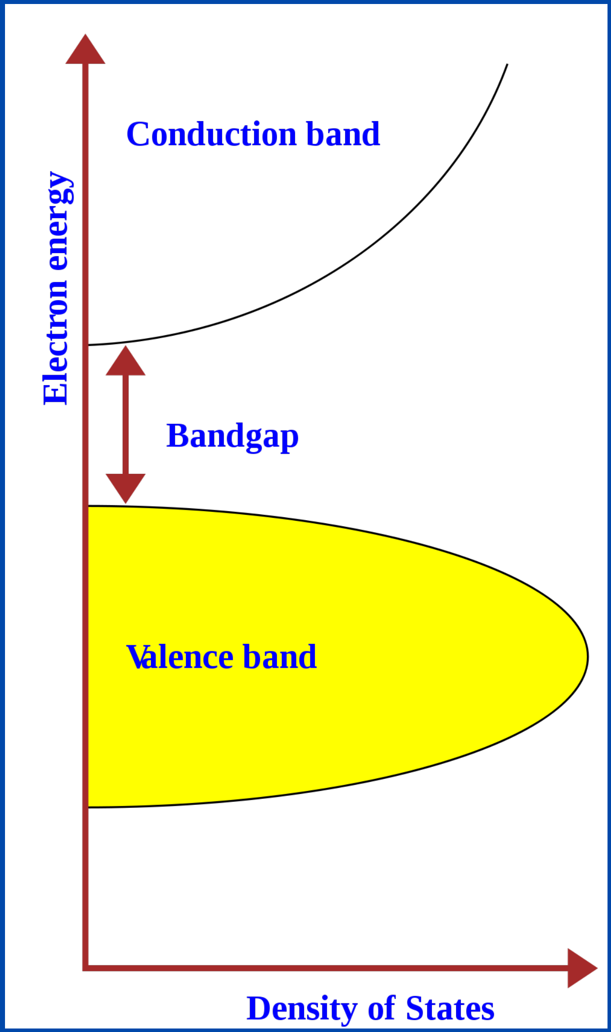
- Photographic plates
  - very low Quantum Efficiency: QE ~ 1 - 4%
  - non-linear response
  - very large areas, very small "pixels" (grains of silver compounds)
  - hard to digitize

- Photomultiplier tubes
  - low QE (10%)
  - no noise: each photon produces cascade
  - linear at low signal rates
  - easily coupled to digital outputs



Modern detectors are based on semiconductors

- In semiconductors and insulators, electrons are confined to a number of specific bands of energy
- “Band gap" = energy difference between top of valence band and bottom of the conduction band
- For an electron to jump from a valence band to a conduction band, need a minimum amount of energy
- This energy can be provided by a photon, or by thermal energy, or by a cosmic ray
- Vacancies or holes left in valence band allow it to contribute to electrical conductivity as well
- Once in conduction band, electron can move freely



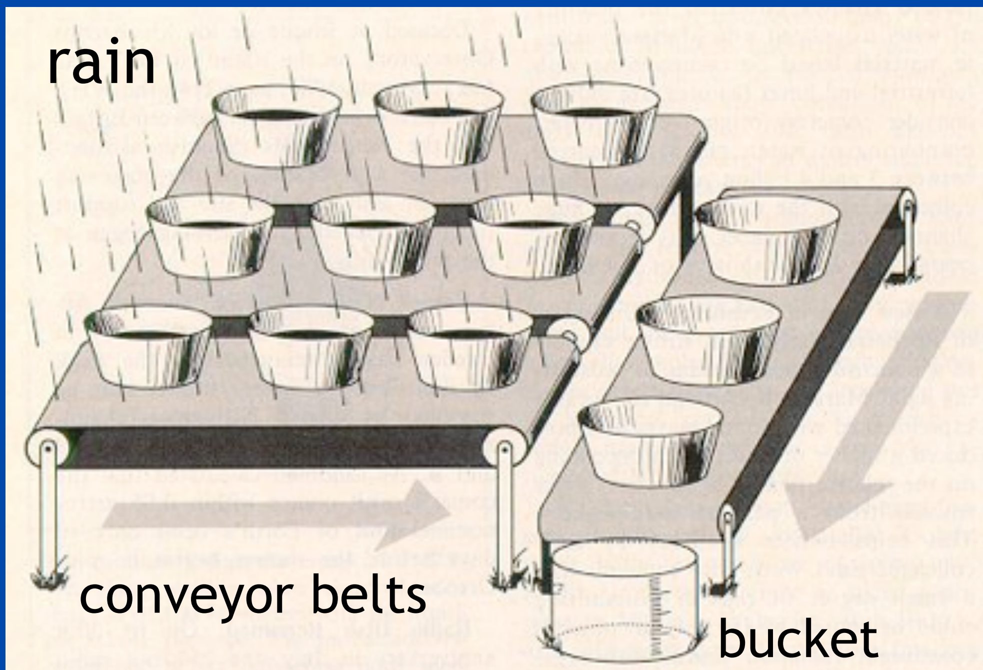
Bandgap energies for commonly used detectors

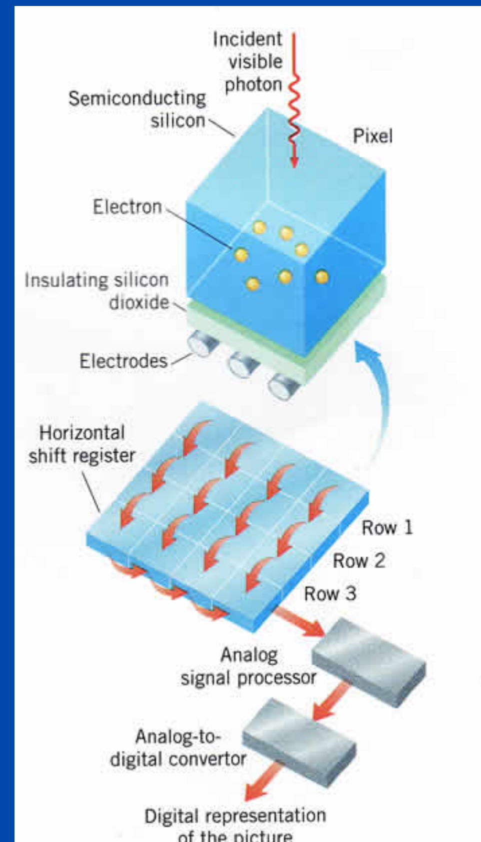
- **If the forbidden energy gap is E_G there is a cut-off wavelength beyond which the photon energy (hc/λ) is too small to cause an electron to jump from the valence band to the conduction band**

Material	Symbol	E_{gap}	λ_{cutoff}
Silicon	Si	1.12 eV	1.1 μm
Indium antimonide	InSb	0.23 eV @ 77K	5.4 μm
Mercury cadmium telluride	$Hg_xCd_{1-x}Te$	0.5 eV (x=0.55)	2.5 μm

Credit: Ian McLean

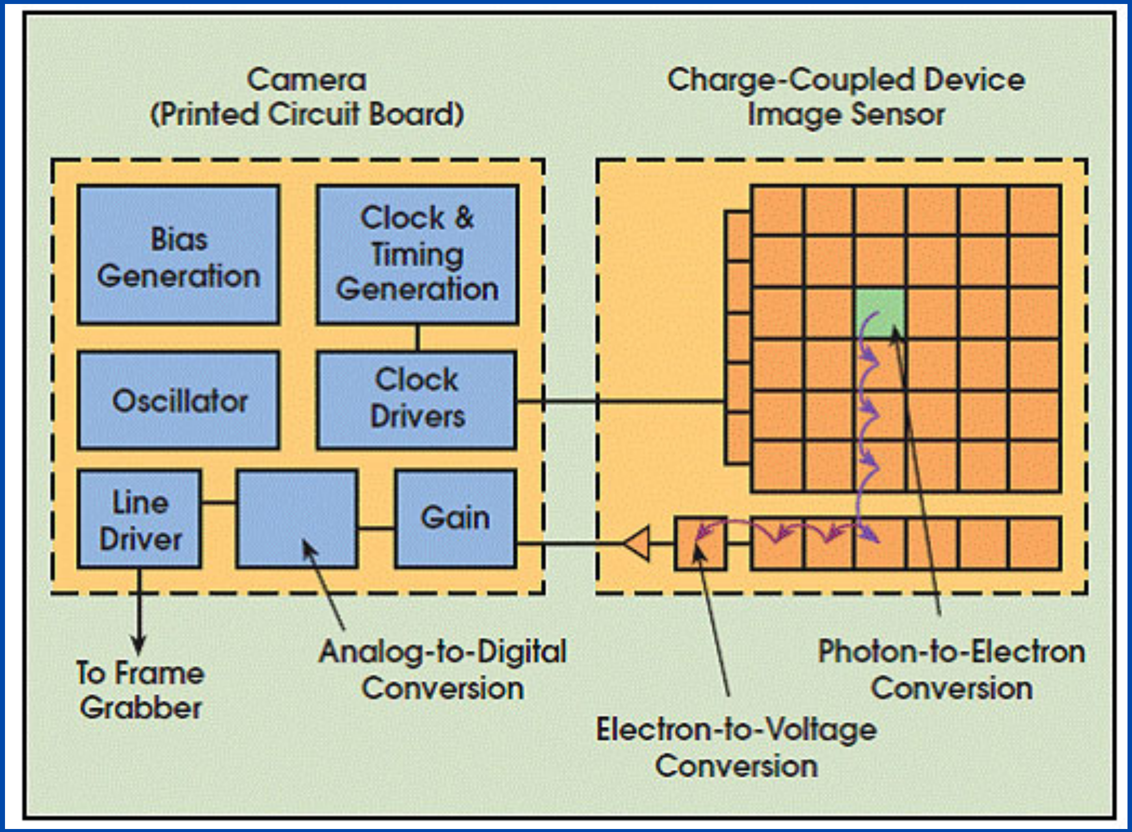
CCD transfers charge from one pixel to the next in order to make a 2D image





- By applying "clock voltage" to pixels in sequence, can move charge to an amplifier and then off the chip

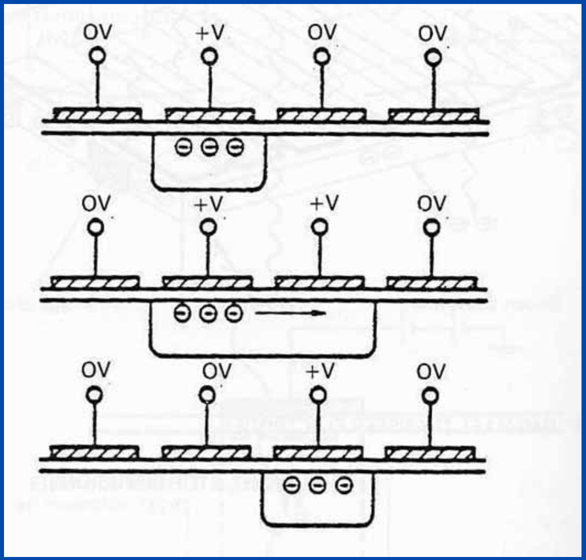
Schematic of CCD and its read-out electronics



- **"Read-out noise" injected at the on-chip electron-to-voltage conversion (an on-chip amplifier)**

CCD readout process: charge transfer

- Adjusting voltages on electrodes connects wells and allow charge to move
- Charge shuffles up columns of the CCD and then is read out along the top
- Charge on output amplifier (capacitor) produces voltage



Video animation:
https://www.youtube.com/watch?v=PoXinWleWns

Modern detectors: photons → electrons → voltage → digital numbers

- With what efficiency do photons produce electrons?
- With what efficiency are electrons (voltages) measured?
- Digitization: how are electrons (analog) converted into digital numbers?
- Overall: What is the conversion between photons hitting the detector and digital numbers read into your computer?

Primary properties of detectors

- Quantum Efficiency *QE*: Probability of detecting a single photon incident on the detector
- Spectral range (*QE* as a function of wavelength)
- "Dark Current": Detector signal in the absence of light
- "Read noise": Random variations in output signal when you read out a detector
- Gain *g* : Conversion factor between internal voltages and computer "Data Numbers" DNs or "Analog-to-Digital Units" ADUs

Secondary detector characteristics

- Pixel size (e.g. in microns)
- Total detector size (e.g. 1024 x 1024 pixels)
- Readout rate (in either frames per sec or pixels per sec)
- Well depth (the maximum number of photons or photoelectrons that a pixel can record without "saturating" or going nonlinear)
- Cosmetic quality: Uniformity of response across pixels, dead pixels
- Stability: does the pixel response vary with time?

CCDs are the most common detector for wavefront sensors

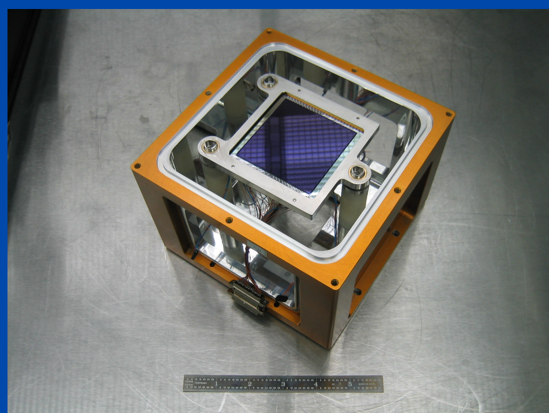
- Can be read out fast (e.g., every few milliseconds so as to keep up with atmospheric turbulence)
- Relatively low read-noise (a few to 10 electrons)
- Only need modest size (e.g., largest today is only 256x256 pixels)

CCD phase space

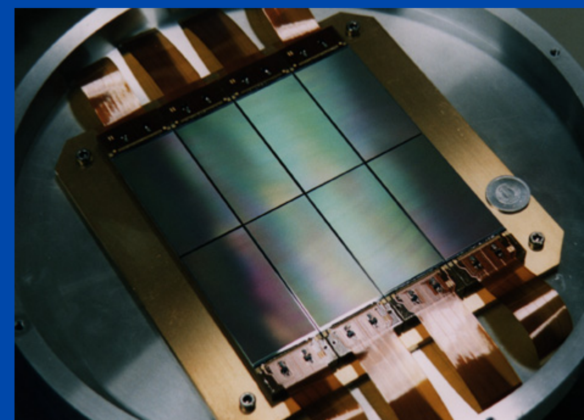
- CCDs dominate inside and outside astronomy
  - Even used for x-rays
- Large formats available (4096x4096) or mosaics of smaller devices. Gigapixel focal planes are possible.
- High quantum efficiency 80%+
- Dark current from thermal processes
  - Long-exposure astronomy CCDs are cooled to reduce dark current
- Readout noise can be several electrons per pixel each time a CCD is read out
  - » Trade high readout speed vs added noise

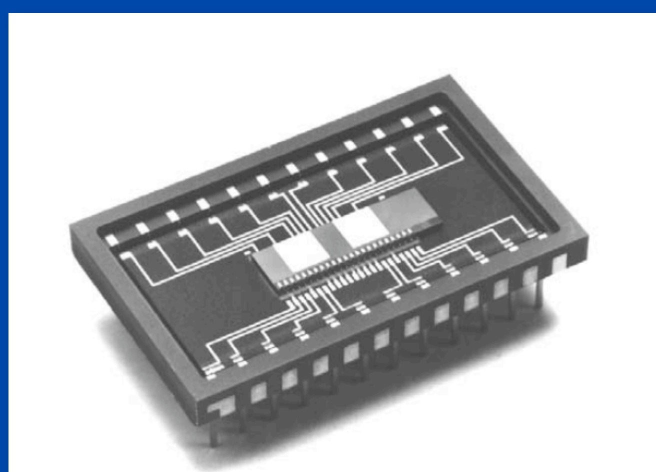
What do CCDs look like?

Carnegie 4096x4096 CCD
Slow readout (science)



Subaru SuprimeCam Mosaic
Slow readout (science)

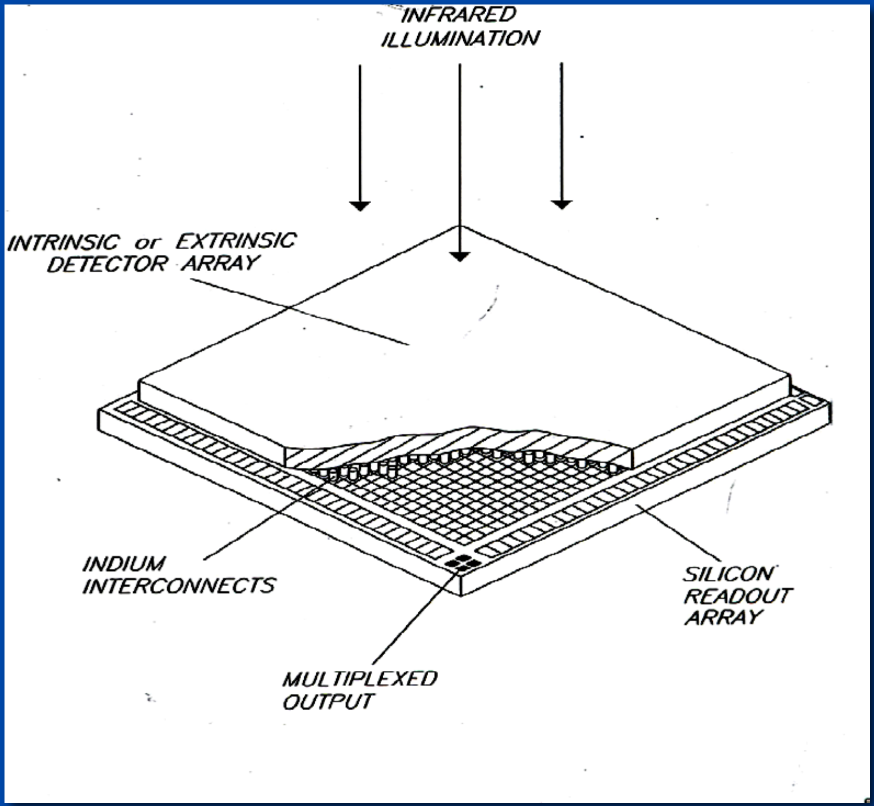




E2V 80 x 80
fast readout for
wavefront sensing

Infrared detectors

- Read out pixels individually, by bonding a multiplexed readout array to the back of the photo-sensitive material
- Photosensitive material must have lower band-gap than silicon, in order to respond to lower-energy IR photons
- Materials: InSb, HgCdTe, ...



Types of noise in instruments

- Every instrument has its own characteristic background noise
  - Example: cosmic ray particles passing thru a CCD knock electrons into the conduction band
- Some residual instrument noise is statistical in nature; can be measured very well given enough integration time
- Some residual instrument noise is systematic in nature: cannot easily be eliminated by better measurement
  - Example: difference in optical path to wavefront sensor and to science camera
  - Typically has to be removed via calibration

Statistical fluctuations = "noise"

- Definition of variance:

$$\sigma^2 \equiv \frac{1}{n}\sum_{i=1}^{n}\left(x_i - m\right)^2$$

where *m* is the mean, *n* is the number of independent measurements of *x*, and the x_i are the individual measured values

- If x and y are two random independent variables, the variance of the sum (or difference) is the sum of the variances:

$$\sigma_{tot}^2 = \sigma_x^2 + \sigma_y^2$$

Main sources of detector noise for wavefront sensors in common use

- Poisson noise or photon statistics
  - Noise due to statistics of the detected photons themselves
- Read-noise
  - Electronic noise (from amplifiers) each time CCD is read out
- Other noise sources (less important for wavefront sensors, but important for other imaging applications)
  - Sky background
  - Dark current

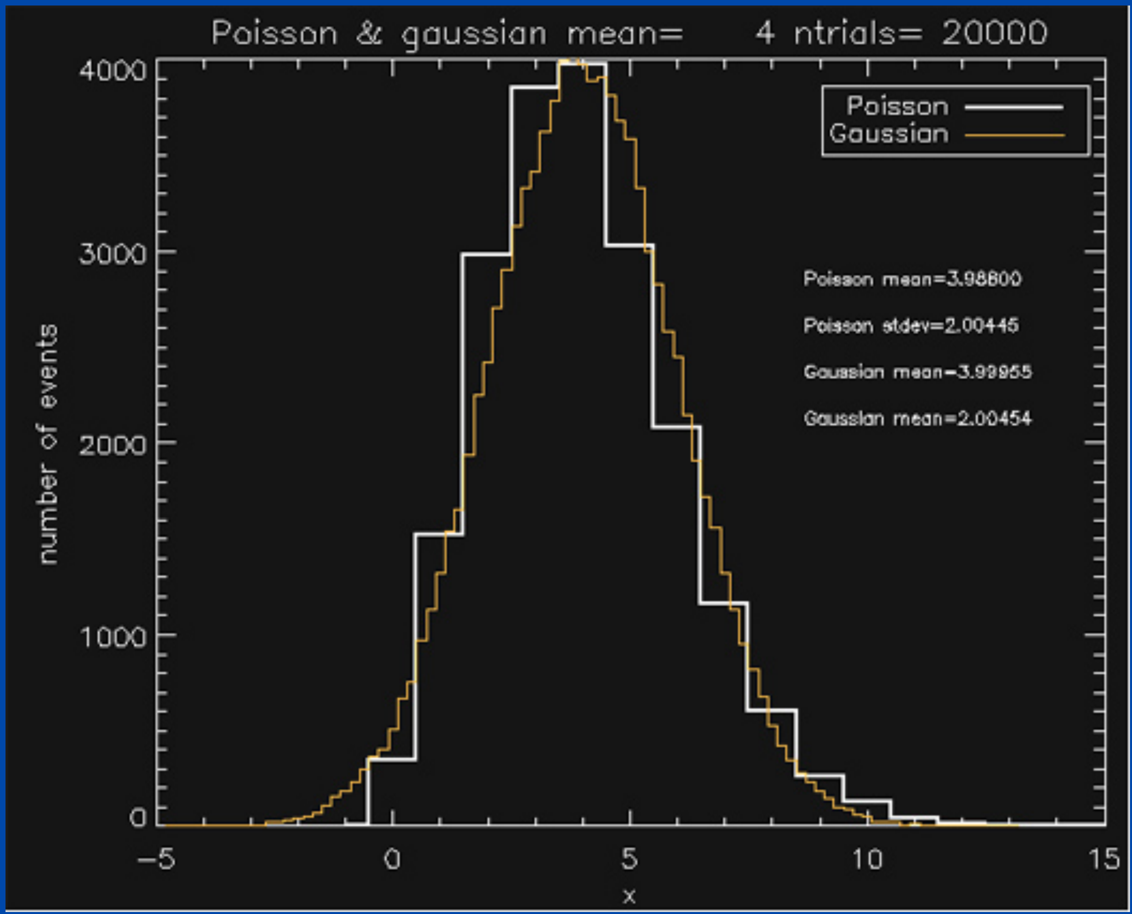
Photon statistics: Poisson distribution

- CCDs are sensitive enough that they care about individual photons
- Light is quantum in nature. There is a natural variability in how many photons will arrive in a specific time interval *T* , even when the average flux *F* (photons/sec) is fixed.
- We can't assume that in a given pixel, for two consecutive observations of length *T*, the same number of photons will be counted.
- The probability distribution for *N* photons to be counted in an observation time *T* is

$$P(N|F,T) = \frac{(FT)^N e^{-FT}}{N!}$$

Properties of Poisson distribution

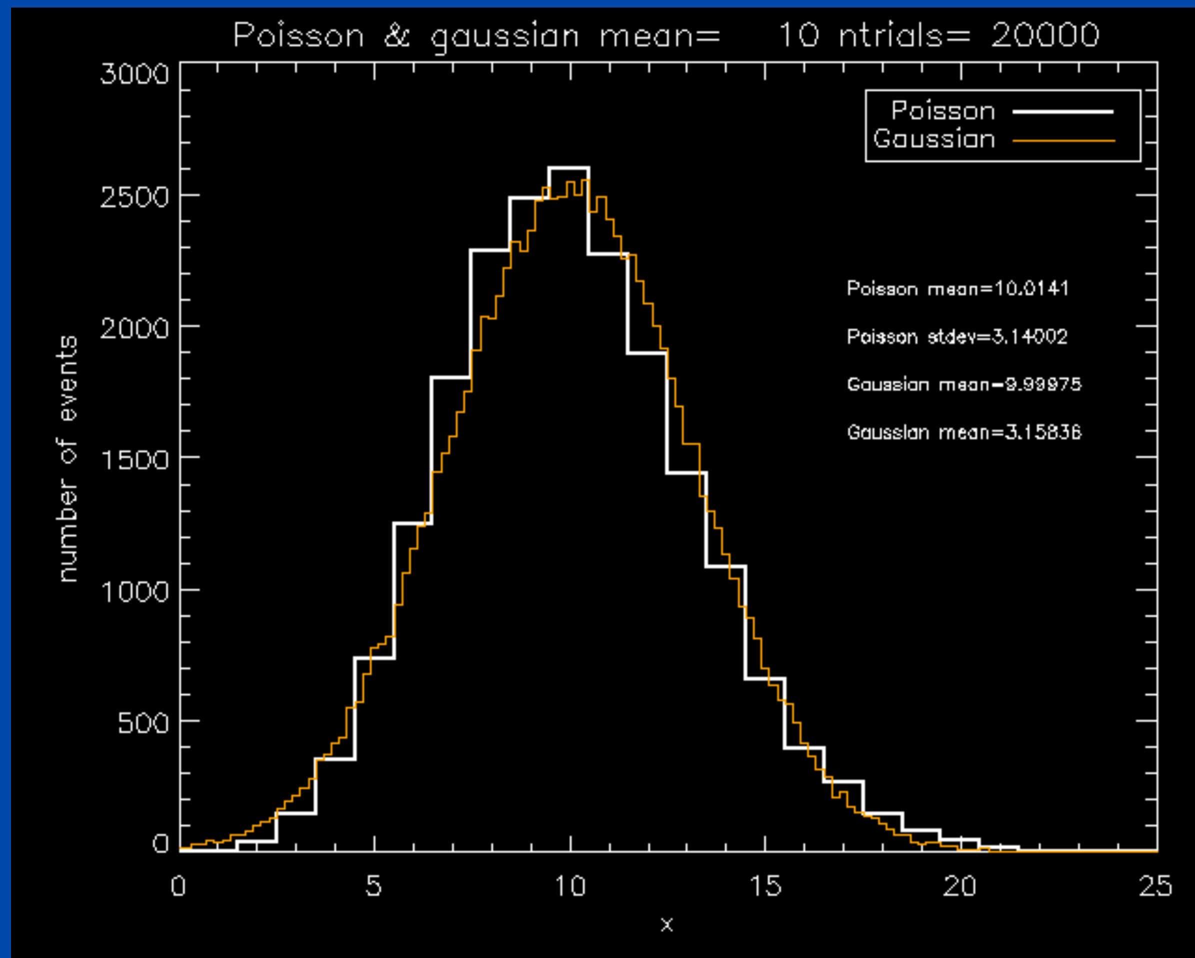
- Average value = *FT*
- Standard deviation = $(FT)^{1/2}$
- Approaches a Gaussian distribution as *N* becomes large



Horizontal axis: *FT*

Credit: Bruce Macintosh

Properties of Poisson distribution

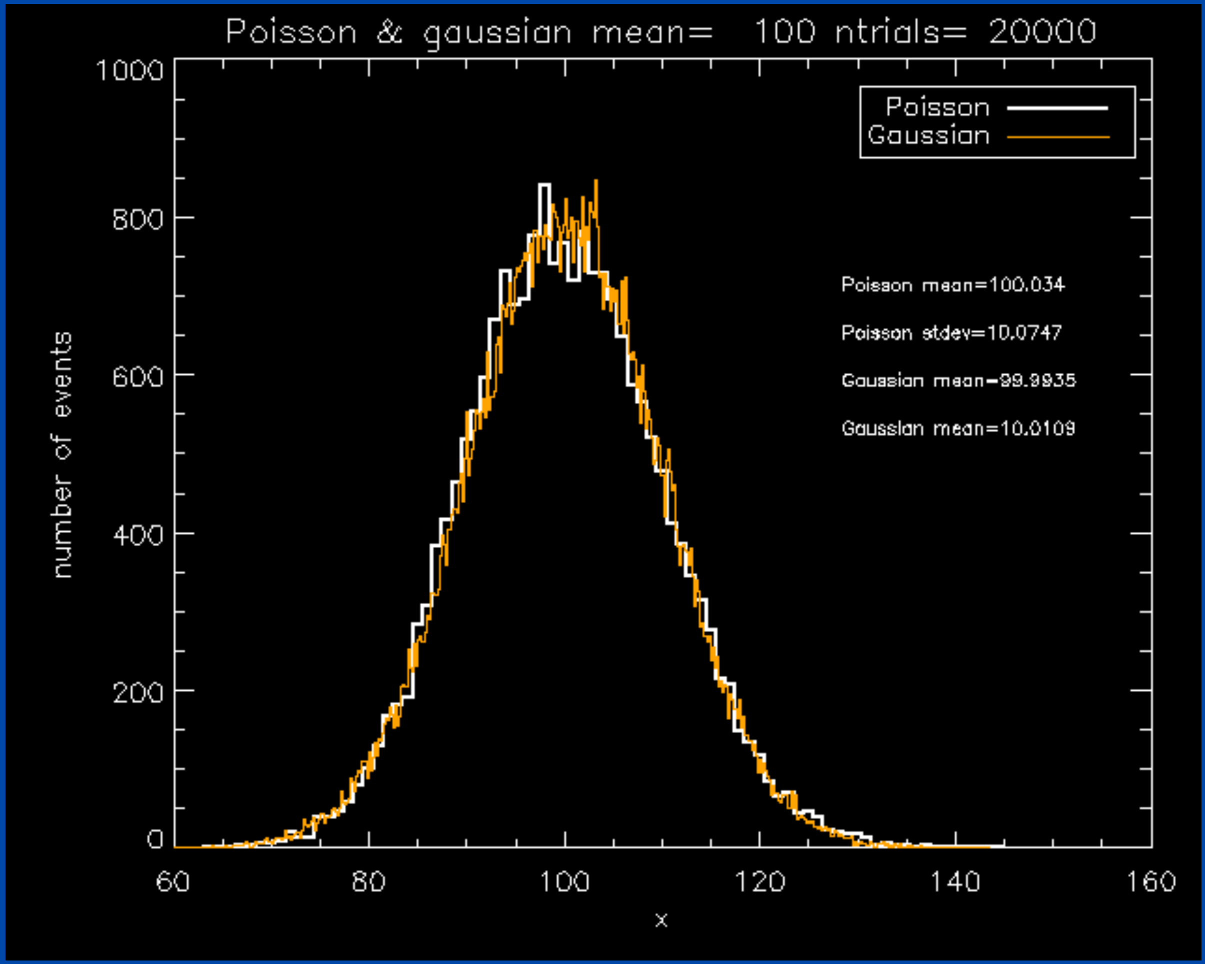


Horizontal axis: ***FT***

Credit: Bruce Macintosh

Properties of Poisson distribution

- When < *FT* > is large, Poisson distribution approaches Gaussian
- Standard deviations of independent Poisson and Gaussian processes can be added in quadrature



Horizontal axis: *FT*

Credit: Bruce Macintosh

How to convert between incident photons and recorded digital numbers ?

- Digital numbers outputted from a CCD are called Data Numbers (DN) or Analog-Digital Units (ADUs)
- Have to turn DN or ADUs back into microvolts → electrons → photons to have a calibrated system

$$\text{Signal in DN or ADU} = \left(\frac{QE \times N_{photons}}{g} \right) + b$$

where *QE* is the quantum efficiency (what fraction of incident photons get made into electrons), *g* is the photon transfer gain factor (electrons/DN) and *b* is an electrical offset signal or bias

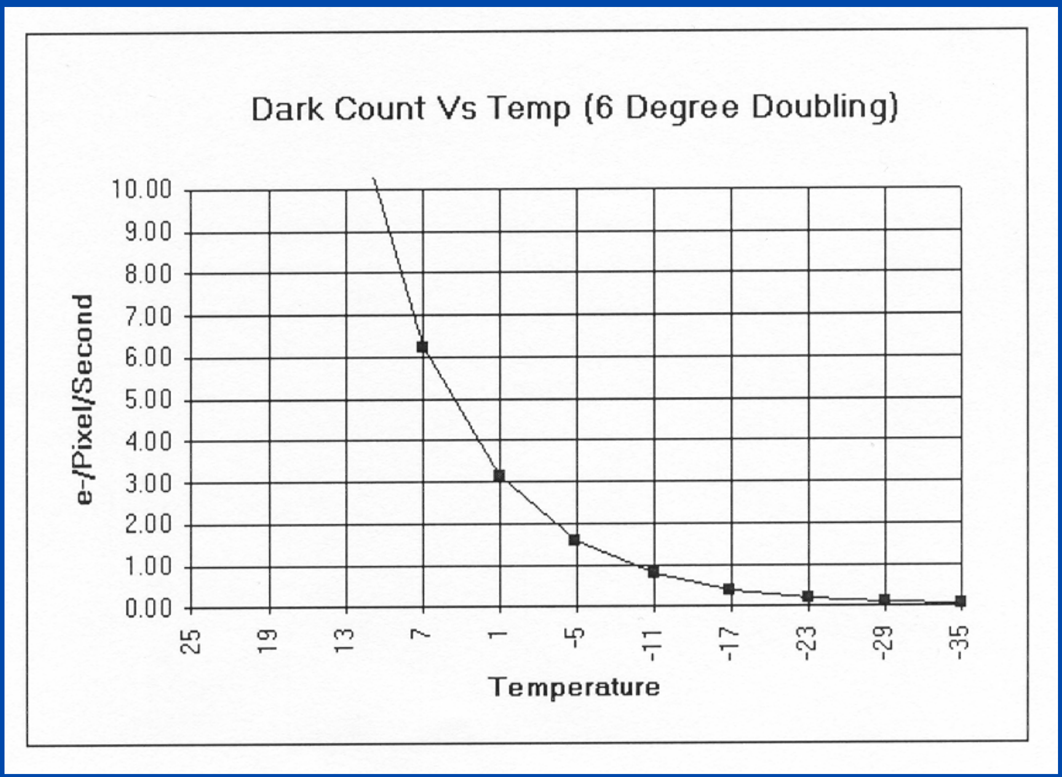
To calculate SNR, look at all the various noise sources

- Wisest to calculate SNR in electrons rather than ADU or magnitudes
- Noise comes from Poisson noise in the object, Gaussian-like readout noise *RN* per pixel, Poisson noise in the sky background, and dark current noise *D*
- Readout noise: $\sigma_{RN}^2 = n_{pix} RN^2$

  where n_{pix} is the number of pixels and *RN* is the readout noise
- Photon noise: $\sigma_{Poisson}^2 = FT = N_{photo-electrons}$
- Sky background: for B_{Sky} e^-/pix/sec from the sky, $\sigma_{Sky}^2 = B_{Sky} T$
- Dark current noise: for dark current *D* (e^-/pix/sec)

$$\sigma_{Dark}^2 = D n_{pix} T$$

Dark Current or Thermal Noise: Electrons reach conduction bands due to thermal excitation



Science CCDs are always cooled (liquid nitrogen, dewar, etc.)

Credit: Jeff Thrush

Total signal to noise ratio

$$SNR = \frac{FT}{\sigma_{tot}} = \frac{FT}{\left[FT + (B_{sky} n_{pix} T) + (D n_{pix} T) + (RN^2 n_{pix}) \right]^{1/2}}$$

where *F* is the average photo-electron flux, *T* is the time interval of the measurement, B_{Sky} is the electrons per pixel per sec from the sky background, *D* is the electrons per pixel per sec due to dark current, and *RN* is the readout noise per pixel.

Some special cases

- *Poisson statistics*: If detector has very low read noise, sky background is low, dark current is low, SNR is

$$SNR_{Poisson} = \frac{FT}{\sqrt{FT}} = \sqrt{FT} \propto \sqrt{T}$$

- *Read-noise dominated*: If there are lots of photons but read noise is high, SNR is

$$SNR_{RN} = \frac{FT}{\left[RN^2 n_{pix} \right]^{1/2}} = \frac{FT}{RN\sqrt{n_{pix}}} \propto T$$

If you add multiple images, SNR ~ $(N_{images})^{1/2}$

Typical noise cases for astronomical AO

- Wavefront sensors
  - Read-noise dominated: $$SNR_{RN} = \frac{FT}{RN\sqrt{n_{pix}}}$$
- Imagers (cameras)
  - Sky background limited: $$SNR_B = \frac{FT}{\left[B_{sky} n_{pix} T\right]^{1/2}} = \frac{F\sqrt{T}}{\left[B_{sky} n_{pix}\right]^{1/2}}$$
- Spectrographs
  - Either sky background or dark current limited:

$$SNR_B = \frac{F\sqrt{T}}{\left[B_{sky} n_{pix}\right]^{1/2}} \quad \text{or} \quad SNR_D = \frac{F\sqrt{T}}{\left[D\ n_{pix}\right]^{1/2}}$$

Part 3: Class Projects (go to second ppt)